MAT 7381 Solution 6.40, 6.41, 6.42

$$SCR = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij}-\bar y)^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i}\left((y_{ij}-\bar y_i)+(\bar y_i-\bar y)\right)^2$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij}-\bar y_i)^2 + \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar y_i-\bar y)^2 + 2\sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar y_i-\bar y)(y_{ij}-\bar y_i)$$

$$= SCR + \sum_{i=1}^{k} n_i(\bar y_i-\bar y)^2 + 2\sum_{i=1}^{k}(\bar y_i-\bar y)\underbrace{\sum_{j=1}^{n_i}(y_{ij}-\bar y_i)}_{=0}$$

$$= SCE + SCR$$

6.41 $$\sum_{i=1}^{k} n_i(\bar y_i-\bar y)^2 = \sum_{i=1}^{k} n_i(\bar y_i^2+\bar y^2-2\bar y\bar y_i)$$

$$= \sum_{i=1}^{k} n_i\bar y_i^2 + \bar y^2\sum_{i=1}^{k} n_i - 2\bar y\sum_{i=1}^{k} n_i\bar y_i$$

$$= \sum n_i\bar y_i^2 + n\bar y^2 - 2\bar y(n\bar y) \quad \left(\text{car } \bar y = \frac{1}{n}\sum n_i\bar y_i\right)$$

$$= \sum n_i\bar y_i^2 - n\bar y^2$$

6.42 $\bar y_1, \dots, \bar y_k$ sont des variables aleatoires indépendantes de loi $\bar y_i \sim N(\mu_i; \sigma^2/m)$, et $SCE = \sum m(\bar y_i-\bar y)^2$.

$\dfrac{SCE}{\sigma^2} = \dfrac{z'Cz}{\sigma^2/m}$, où $z' = [\bar y_1, \dots, \bar y_k]$, $z \sim N(\underset{\sim}{\mu}, \frac{\sigma^2}{m}I)$.

Utilisant la condition $A\Sigma A = A$ pour que $y'Ay$ soit de loi χ^2, on conclut que $\dfrac{SCE}{\sigma^2} \sim \chi^2_{k-1}(\lambda)$, où $\lambda = \dfrac{\underset{\sim}{\mu}'C\underset{\sim}{\mu}}{\sigma^2/m}$. $\lambda = 0$ ssi $C\underset{\sim}{\mu} = \underset{\sim}{0}$, donc si et seulement si $\underset{\sim}{\mu} = \text{cte}\ \underset{\sim}{e}$.